Communication Ability in Resolving Open-Ended Problems in Mathematics

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Abstract

Open-ended problem solving is problem solving that presents a problem that has more than one correct answer or has more than one correct settlement method. The use of problem solving is not far related to communication. Communication ability is very important ability in problem solving. The purpose of research was to know student's communication abilities in solving open-ended problems with the material equation and quadratic function. The type of the research is descriptive qualitative research which involving two students that is student with high communication ability and student with low communication ability from one of SMPN Wonogiri. Data collected through the results of written test and interview the used triangulation methods to validate the data. Based on research analysis show that: (1) written communication of the students in solving problems have different solutions; (2) the student with high communication ability is better at open-ended problem solving than the student with low communication ability. This shows that students' high communication ability have more than one solution or answer especially in calculation problems, they have various solution to explain detail answer.

Keywords: *Communication Ability; Open-Ended Problem Solving; Mathematics*

Introduction

Mathematics learning focus on solving problems in the form of diagrams, graphs, picture and tables to gain information which outlined in an article or oral. Problem solving is an important part of the mathematics curriculum in the learning process and qustions solving, the students are required to explain the way in of process answering in solving the problems. One of problem in mathematics is mathematics open problems. According to Foong states that *open-problems* is a problem which has not a procedure to get the correct solution and is not structured correctly.

In this research, the researchers focus on mathematics problems solving in the form of open-ended. According to Hino (2007) open-ended problems is a problem which concluded so it has some correct answers. Problems that have more than one correct answer or have open-ended characteristic can increase activity during student learning. Furthermore, Nohda (2000) assumes that open-ended approach is very important for every student to have the freedom in improving problem solving ability appropriate with their interests and abilities. However, during the mathematics learning is dominated by teachers, where the students are never or rare asked the explanation of the source they get the answer. Mathematics

learning suggest that the essence of mathematics learning is practice in estimating, investigating and examining all aspects of problem solving (NCTM, 1989). The students must be given the chance to find the solution of problems to create new problems by modifying the condition of problem that is given. When getting information that is given n the problem, the students can communicate the information in the form of a story or written text so the explanation is suitable with what is got. However, in fact the mathematics communication of junior high school students in Indonesia is classified still relatively poor. In addition to another obstacle, such as the students do not understand about the material being taught and the lack of reading skill which the students have are also become one of factors the mathematical communication development.

Based on the previous description, communication has important role in problem solving, so the researchers are interested to conduct the research related to communication to solve open-ended problem in mathematics.

1. Communication Ability

The process of learning is influenced by the changes of individual behavior through interaction with environment. According to Deni (2011: 9) the active process of individual through the interaction experience causes changes in behavior permanently. Symbolic language is used in communicating the ideas or mathematics concept that make possible everyone to understand the meaning of a statement so accurate and appropriate communication is realized. Communication is dynamic process where some people try to share their internal problems with others through the use of symbols (Larry, 2010: 18). Communication also needs several convention of linguistics, psychological which are planted in human communication itself (Erton, 2018), because communication is fundamental ability which must be possessed by the students so it needs to be developed in mathematics learning. Barrody (in Lim and Chew, 2007) states two reasons that communication becomes one of the focuses in mathematics learning. First, basically mathematics is a language. Mathematics is not only a thinking tool that help students to find patterns, problem solving and draw conclusion. But also is tools to communicate the students' thought about clear, appropriate and brief ideas. Second, mathematics learning is a social activity which involve at least two subjects, thait is the teacher and student. In learning must be able to help the students in communicating the mathematics idea through five aspects of communication, thait is discussion, listening, reading, representing, and writing, Baroody (in Qohar, 2007).

According to research conducted by Sokoine (2015) communication ability which felt by most of respondents are very important for communication skills acquisition that is needed in their life. Study communications in mathematics helps interaction development and idea expression in classroom because the students learn in active atmosphere. Effective communication ability can be interpreted as the occurrence the similarity of meaning that the speaker wants to explain with meaning that the listener understands (Dixon, 2012: 5). Therefore, communication abilities must be developed from early age, one of them is developed in mathematics learning. Communication is very important part in mathematics learning. This is supported by the opinion of Asikin (in Darkasyi, M., Johar, R & Ahmad, A, 2014) that the role of communication in mathematics learning are: (1) mathematical communication can be utilized in variety perspectives, help to sharp the way of students' thinking and to hone students ability in seeing something relationships of mathematics; (2) communication is a tool to measure the development and reflect of understanding the students' mathematics; (3) through communication the students can build and sequence their mathematical thinking; (4) communication between the students in mathematics learning is very important to developing problem solving, and improving social skills; (5) "talking and writing" can be used as meaningful tool to create mathematical community inclusively. Clark & Jemmifer (2005) states that to develop communication ability can be given four strategies, that is: (1) giving sufficient tasks (to make the students and discussion groups more active), (2) creating conducive environment in order to the students can express their ideas freely, (3) directing the students to explain and give arguments on the results which given and ideas that they thinking, (4) directing the students to active in

processing various ideas and concepts. Based on expert opinion it can be concluded that the definition of communication ability is the process in sharing ideas and clarify understanding by conveying information that is got to others including symbols, images, graphics, and diagrams through written or oral.

Indicators of the students' mathematical communication ability according to NCTM (in Fachrurazi: 2011) that is: a) the express mathematical ideas through oral, written, and draw them visually, b) the understanding, interpreting, and evaluating mathematical ideas orally, written, c) using technical term   s, mathematical notations and describe the relationships with situation models.

Indicators of communication ability that is used in this research is written text and drawing text which are explained as follows:

Table 1. Indicators of Communication Ability in Open-Ended Problem Solving

The steps of problems solving	Communication indicators in open-ended problem solving
1. Understand the problem 2. Arrange the plan of solution 3. Implement the plan of solution 4. Re-check	*a.* Written text 1) Understand the problem by making statement about the material that has been lerabed 2) Give the answer by using own language
1. Understand the problem 2. Arrange the plan of solution 3. Implement the plan of solution 4. Re-check	*b.* Drawing text. 1) Reflect the information that has been obtained in the form of pictures and tables 2) Interpret the answer suitable with the information that has been obtained

Ali Mahmudi (2009) states that good communication processes, especially using open problem in mathematics learning, can support the students to develop ideas and mathematical knowledge.

2. Open-Ended Problem Solving

Monaghan et al. (2009) divided mathematical open-problem into two, that is open-ended which has more than one solutions or answers, whereas open-start mathematical problems have closed answer or only has one final answer. Hellstrom state about the differences of both problems as follows:" Based one or different strategies (start) and one or different answer (ends), open start problem would be many strategies-one answer test items". The meaning of that statement is open-ended problem has many solutions, but open-start problem has many strategies in problem solving. In the history of education, problem solving is one of the important educational goals for teachers or parents who is intended in order to their students have problem solving ability. One of way in problem solving can be through the action and make new approach in problem situation (Cai & Cifarelli, 2005). Individuals are often involved in problem solving processes in school or daily life (Blitzer, 2003) so they have ability to communicate. Problem solving is one of the basic skill that help the student to face real life and is shown by research as one of mathematical communication ability (OECD, 2014).

The problem solving in this research used the steps which has been stated by Polya (1973: 5-19) consist of: (1) understanding the problem is important step in solving a problem. In this step the statement in the question must be understood correctly. The thing to do is state the main parts of the problem, what data is available and what characteristics are needed in solving the problems; (2) devising a plan. In this step, need a ability to see relationship between the data and conditions which existed or unknown data, the subject is hoped to make mathematical model which can be solved by using mathematical rules; (3)

carrying out the plan. Every step in the process must be really thorough and must be proven the validity. In this step, mathematics skill that is used is counting skill in quadratic equations; (4) recheck. In this step certainly the process and the results is suitable with what is desired in the questions or not. If the results do not match with what is requested in questions, so it need to recheck every step that has been done to get the result which suitable with questions given, and see others possibilities that can be done to solve the problems. This step requires subject to predict the right strategy to solve the problem that be faced. Oliveira and Viseu (2012) states that communication in mathematics learning through open-ended is more effective to do with peer students. It is expected that the students get the wide experience in using knowledge and skills that they have to apply in problem solving especially related to daily life.

Method

This research was conducted by using descriptive qualitative research. Descriptive research that explain the situations which be given clearly and detail (Büyüköztürk et al., 2011). This research aims to describe the case and describe the parts and analyze until reach a conclusion which suitable with research objectives (Cohen, et al, 2000). This method is often used in research, because it uses check in detail the students' communication in problem solving. The stage of sampling in this research is determine subject which have high communication ability and low communication ability in grade IX who is given initial subject A and subject B. Then the data collection was taken from the results of written test about equations and quadratic fuctions questions and interview transcripts. In this case, semi-structured interviews which are held with participants and determined through the questions that has been prepared. Data collected through the result of written test and interviews then used triangulation methods to validate data. Furthermore, the data which has been collected analyze based on mathematical communication. Then it measured based on the indicators of communication that is used. The instruments consist of 2 numbers with equations and quadratic functions material. Question number 1 about finding quadratic equations and number 2 looking for table of value x and $f(x)$ and draw graph in quadratic functions. Next step is doing review about the result of written test which statement that explained by the subject during the interviews until reach a conclusion.

Results and Discussion

The finding that revealed is suitable with the results of written tests and interviews of subject A and subject B, then it analyzed and compared so find the valid data. After that, data is classified according to high communication ability and low communication ability. Communication ability have two indicators that is written text and drawing text. The explanation of both indicators are presented in table 1

Table 2. Communication Ability About Written Text

The explanation about indicators of written text in open-ended problem solving of question number 1 part a)

Valid data of subject A	Valid data of subject B
1. Subject A can understand the problem by determining the value that was asked in questions	1. Subject B can understand the problems by determining the value that was asked in questions
Example : subject A and subject B can determine the value *a, b,* and *c* after understanding the command of the questions from equation $x^2 - 7x - 18 = 0$	
2. Subject A arrange the plan of solution by finding the suitable formula to count two variables that is known	2. Subject B arrange the plan of solution by finding the suitable formula to count two variables that is known
Example : Subject A and B can solve problem solving $X_1 + X_2$ with the formula which they	

known, that is $-\frac{b}{a}$ for solution of subject A , whereas subject B use formula $-b \pm \frac{\sqrt{b^2 - 4.a.c}}{2.a}$, solving problems $X_1 . X_2$ by using fraction $\frac{c}{a}$ for subject A and subject B use formula $X_1 . X_2 = \frac{1 \pm \sqrt{b^2 - 4.a.c}}{4}$ and problems $\frac{1}{X_1} + \frac{1}{X_2}$ for subject A use formula $\left(\frac{X_1 + X_2}{X_1 . X_2} \right)$

3. Subject A count the result of problem solving correctly	3. Subject B is not yet appropriate and correct in counting the result of problem solving

Example : Subject A count the result of $X_1 + X_2 = -\frac{7}{1}$, subject B with the result of counting 7 is uncorrect in counting positive mark and introgative mark. Furthermore the counting of problem solving of $X_1 . X_2$ subject A get result $-\frac{18}{1}$, whereas subject B get result $0,5$ which is still wrong in solve the problems and the last problem solving of $\frac{1}{X_1} + \frac{1}{X_2}$ the result of counting subject A $\frac{-7}{-18}$ is correct, whereas subject B $-\frac{1}{15}$ and the answer is still uncorrect

4. Subject A re-check the answer by reading the question and understand the information that is got then find the formula and then count every problem and stated that it is appropriate and correct	4. Subject B re-check the answer and actually in determining formula is still less appropriate and cause a wrong in counting final result

Based on table 2 which explain about Written text communication in detail between subject A and subject B can be concluded that subject with high communication ability understand the problems by making statement about material that has been given so in way of solving the subject understand what they will write correctly in their solving and make the formula which will be used correctly, whereas the subject with low communication ability make mistake in making a solution to answer the question, such as use the formula that is still wrong so in counting the solutions is less appropriate.

The explanation about indicators of Written text in open-ended problem solving question number 1 part b.

Table 3. Communication Ability About Written text

Data Valid subject A	Data Valid subject B
1. Subject A can understand the problems by determining the value that is asked in question	1. Subject A can understand the problems by determining the value that is asked in question

Example : Subject A and Subject B can determine value *a, b,* and *c* after understand the command of questions from equal $2x^2 - x - 15 = 0$

2. Subject A arrange formula to solve the problems by counting suitable with the question that is given.	2. Subject B make different formula with subject B

Example : Subject A and B can solve the problems of $X_1 + X_2$ by using formula that is known, that is $-\frac{b}{a}$ for solving subject A, whereas subject B use formula

$-b \pm \frac{\sqrt{b^2 - 4.a.c}}{2.a}$, solve the problems of $X_1.X_2$ by using fraction $\frac{c}{a}$ for subject A and

subject B using formula $X_1.X_2 = \frac{1 \pm \sqrt{b^2 - 4.a.c}}{4}$ and problems of $\frac{1}{X_1} + \frac{1}{X_2}$ for subject A

use formula $\left(\frac{X_1 + X_2}{X_1 \cdot X_2}\right)$

3. Subject A do the plan of solution by counting every number suitable with formula that has been determined, and it is less correct in using mark although the calculation is correct	3. Subject B count the problems that is got by using formula which is made, and cause the result that is got unsuitable and less appropriate

Example : Subject A count the result of $X_1 + X_2 = -\frac{1}{2}$, subject B with the result of calculation $0,5$ is not appropriate in the sum of positive and interogrative sign. Furthermore, the calculation of problem solving $X_1.X_2$ subject A gets $-\frac{15}{2}$, while subject B gets $0,5$ which this result are still wrong in solving the problems and the last problem solving of $\frac{1}{X_1} + \frac{1}{X_2}$ the calculation of subject A is $\frac{-1}{-15}$ which the answer is correct, whereas subject B $\frac{1}{0,5}$ and the answer is still not correct

4. Subject A recheck the answer by reading the questions and understand the information that is obtained then find a formula and then count each problem	4. Subject B recheck the answer and actually in determining formula there is something is still not appropriate and cause mistake in final calculation

Based on table 3 which explain about written text communication in detail between subject A and subject B can be concluded that subject with high communication ability give the answer by using formula that has been learned and then used to solve the problem so it get the correct answer although the use of symbols is still not correct, whereas subject with low communication ability make mistake in answering the questions because when doing it use a formula that is less appropriate and still wrong.

The explanation about the indicators of drawing text in open-ended problem solving (the question number 2).

Table 4. Communication Ability About Drawing Text

Valid data of subject A	Valid data of subject B
1. Subject A was able to understand the problem by determining value that is asked in the questions	1. Subject B was able to understand the problem by determining value that is asked in the questions

Example : Subject A and Subject B was able to make a table of value x and
$f(x) = x^2 - 9x + 18$ with origin area $2 \le x \le 7, x \in R$ (real number)

2. Subject A arrange settlement plan by determining component x with origin area $2 \leq x \leq 7, x \in R$ (real number)	2. Subject B look for the value of component x with origin area $2 \leq x \leq 7, x \in R$ (real number)

Example : Subject A and Subject B mention the component $x = 2,3,4,5,6, and\ 7$, then make table x and $f(x)$,on subject B the result of $f(x)$is not written.

3. Subject A draw graph suitable with value x and $f(x)$ that has been found	3. Subject B draw graph suitable with value x and $f(x)$ that has been determined

Example : Subject A and B was able draw graph that has been got from table of value x and $f(x)$ that has been counted suitable with value x

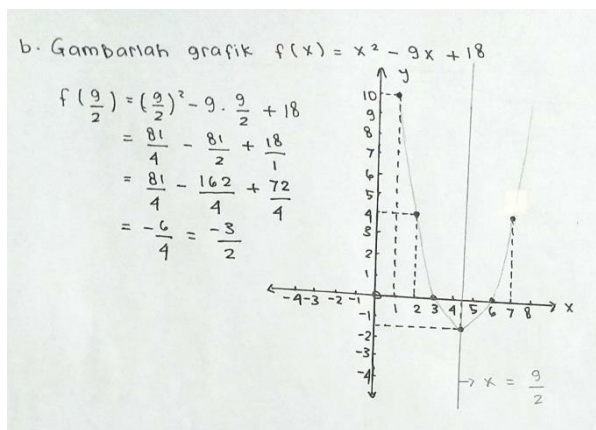| 4. Subject A count the result of problem solving correctly by proving the value of table with the graphical image that has been drawn is suitable	4. Subject B check the result that has been got, and realize that the value of $f(x)$is not yet written

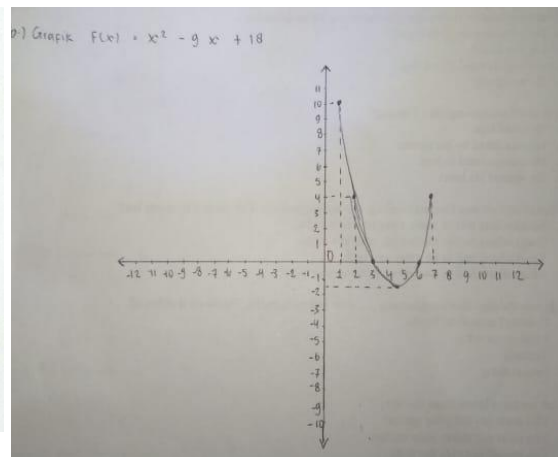Figure 1. the result of the written test subject A number 2

Figure 2. the result of written test subject B number 2

Based on table 4 which explain about drawing text communication in detail between subject A and subject B can be concluded that subject with high communication ability reflect the information that has been got from the questions by making table and then draw the value of equation, give the answer by using idea that has been learned and then applied in solving the problems so they get correct answer although in using symbols is still not correct, whereas subject with low communication ability make mistake in arranging settlement plan and cause errors in answering the question because in doing the question the students do not write the value of f(x) but directly draw the graph of the quadratic function. It is based on opinion (Zalenskiy,2013) that the use of problems with various solutions in order to the students have knowledge and able to solve the problems which still foreign for them. In other words, open-ended problem give a chance for the students in finding the problems with various settlement strategy.

The following are the transcript of interviews with subject A

Explanation:

P　　　: the researcher

S-A　　: subject A

S-B　　: subject B

P　　: what is the material discuss in the question above?
S-A　: quadratic equation and quadratic function
P　　: what did you known and asked about the questions?
S-A　: thing that is known and asked about the questions is determining the quadratic equation and quadratic function
P　　: what is the steps do you take in answering the questions
S-A　: understanding first, then I write the value of a, b, and c in question number 1 and for question number 2 I look for the value of x and f(x) then draw the graph
P　　: are there any difficulties when doing the question?
S-A　: yes, there are. When I determine the value of quadratic equation
P　　: which part?
S-A　: determine the sign of final result from the sum of points b, c, d

The following of interview transcript with students B

P　　: what is the material discuss in the question above?
S-B　: quadratic equation
P　　: what did you known and asked about the questions?
S-B　: thing that is known and asked about the questions is quadratic equation of question and what is asked in the question is the value $a, b, c, X_1, X_2, X_1+X_2, X_1.X_2$ and $\frac{1}{X_1} + \frac{1}{X_2}$.
P　　: are there any difficulties in doing the question?
S-B　: yes, there are. In part b and c number 1.
P　　: what is your conclusion after doing that questions?
S-B　: the conclusion is do mathematics is not difficult and not easy, if we learn deeply what is mathematics we will be easier to answer

Based on that description, it can be known that the students' communication ability in explaining and answering the questions for subject A is good enough and fulfill the indicators, although there are some writing signs are wrong and subject explain that they have difficulties in determining the value so it cause symbol errors. For subject B is not fulfill all indicators, because subject B has the difficulties in planning the problem solving that they get about formulas that have been made, and cause the results of calculation is not appropriate and less correct.

Conclusion

Based on the result of the analysis and discussion about the results of communication ability and interviews with 2 subject, it was concluded that: (1) students' high communication ability fulfill indicators of written text and drawing text, that is shown on the result of written test that the student be able to make statement about the formula that will be used, then apply that formula so it produce the

correct answer for questions number 1 point a, then number 1 point b subject A give the answer according to the formula that has been found although the symbol in final result is not correct, which should be positive, but subject answer with negative sign. In the question number 2 subject A be able to look for the value of f(x) in quadratic function, then subject reflect value x and f(x) into graph and obtain image which is appropriate with the value found. (2) students' low communication ability is not fulfill the indicators of written text and drawing text and the formula is not correct so the answer is less appropriate in question number 1 point a and point b. for number 2 subject does not write the value of f(x) but directly draw the value x and f(x) into graph.

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