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Learning Trajectory on Learning Arcs and Tangent of Circle in the Context of the Rubber Fastening of Several Pipes

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Abstract

Circles are material that is applicable in real life. We often encounter circular surface shapes, such as wheels, wall clocks, coins, and pipe surfaces. This research designs a learning trajectory regarding calculating arc lengths and tangents to circles formed by rubber used in packaging several identical and non-identical pipes through the application of learning using the Indonesian Realistic Mathematics Education (RME) approach. The method used goes through 3 phases, namely: (1) Experimental preparation; (2) Experiments in class; and (3) Retrospective Analysis. This design consists of five stages: (1) calculating the length of the rubber binding between the two pipes; (2) calculating the length of the three pipe rubber ties; (3) calculating the length of the four pipe rubber ties; (4) draw conclusions; and (5) Excercise. The results show that a formula has been found to calculate the length of rubber binding several identical pipes.

Keywords: *Learning Trajectory; Arcs of Circles; Tangents to Circles; Identical Pipes*

Introduction

Mathematics is one of the most important subjects. One of the objectives of mathematics is to equip learners to solve problems that include the ability to understand problems, design mathematical models, solve models or interpret the solutions obtained (mathematical problem solving). Therefore, math is very important for learners to solve everyday problems (Decree of BSKAP No.8, 2022).

Mathematics education at the secondary level is often faced with significant challenges, especially in learning circular material. Empirical facts show that the traditional approach which emphasizes memorizing formulas related to circles is not effective in building a deep conceptual understanding. More emphasis on formulas without understanding the concept can lead to minimal learning outcomes, as well as weaknesses in problem-solving in real-world contexts. This approach often leads to minimal learning outcomes and hinders students' ability to solve real-world problems related to circles (Kajander & Lovric, 2005).

In addressing the challenges of teaching the circle theorem, it is important to focus on conceptual understanding rather than solely emphasizing procedural knowledge. (Fabros & Ibañez, 2023). This finding is supported by research showing that the ability to transfer mathematical properties to unfamiliar contexts is closely related to procedural success and attitudes towards mathematics (Koban, 2015). Student performance in mathematics, at various levels of education, is a focus of attention that requires exploration of instructional resources and teaching methodologies to improve learning outcomes (Changwony et al., 2020).

In conclusion, the challenge of teaching circle theorems in secondary mathematics education demands a paradigm shift toward fostering conceptual understanding. This needs to be accompanied by handling students' emotional aspects and incorporating interactive teaching approaches. In addition, the exploration of effective instructional resources and assessment strategies is an urgent need to improve the quality of mathematics education at various levels of education.

The importance of a strong conceptual understanding of mathematics learning, especially in the circle material, is an important basis for the development of problem-solving skills and the application of mathematics in everyday life. Realistic mathematics education appears as an interesting and innovative alternative to overcome this weakness.

Realistic Mathematics Education (RME) is a learning approach developed in 1971 by mathematicians in the Netherlands. RME was developed based on Freudenthal's view that describes mathematics as a human activity. Mathematics as a human activity is a problem-solving activity, from Finding problems, but also an activity of organizing learning materials. This can be a problem from reality that must be arranged according to a mathematical pattern if the problem from reality must be solved. It can also be a mathematical problem, a new or old result, from yourself or someone else, which must be arranged according to new ideas, to be better understood, in a broader context, or with an axiomatic approach (Gravemeijer & Terwel, 2000).

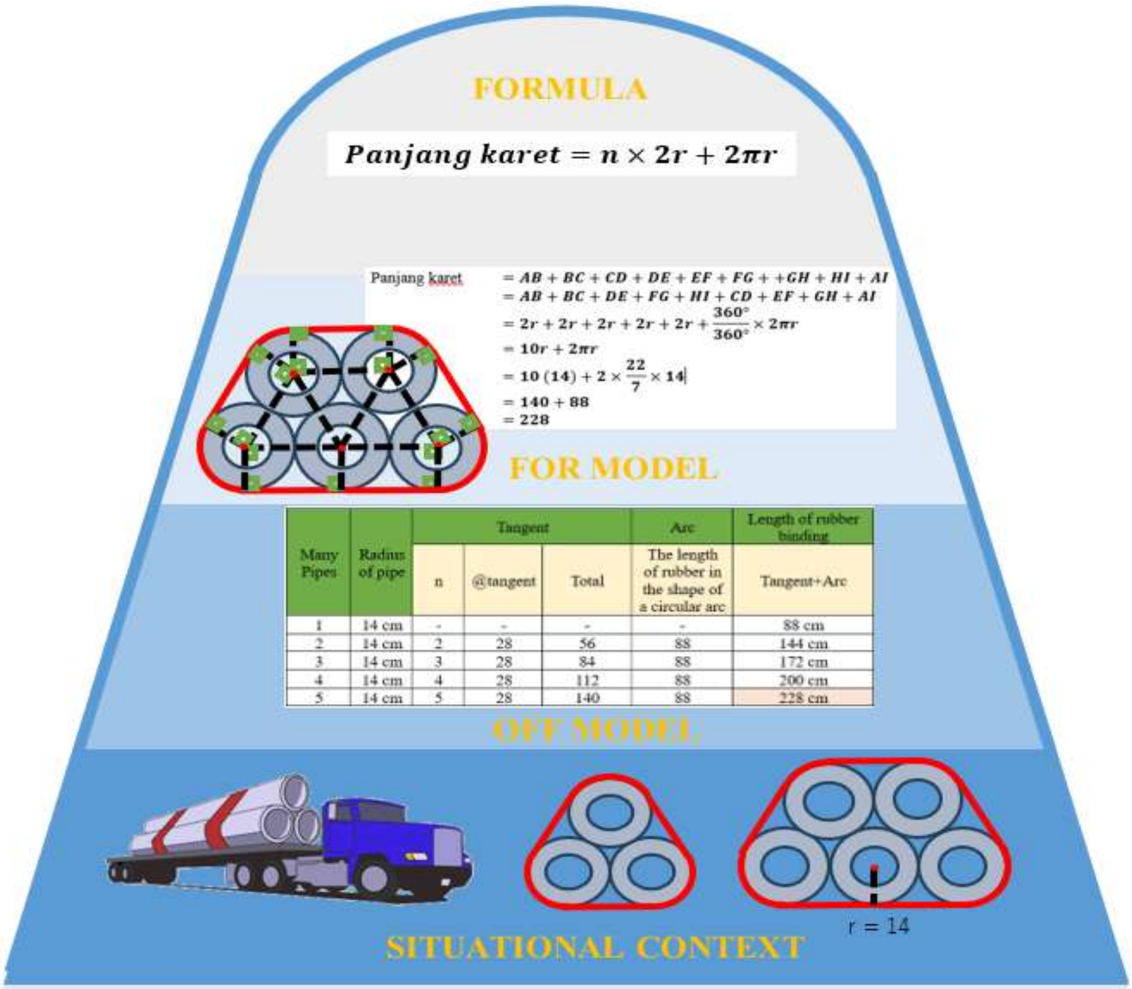
Mathematical activities are distinguished into vertical activities and horizontal activities. Horizontal mathematical activities aim to connect mathematical concepts and methods to real situations, while vertical mathematization occurs entirely in mathematics (Christer Bergsten, 2014).

In Freudenthal's idea, mathematics must be seen as a process of activity, and at the same time the activity will produce a product. Then, to integrate the two, Freudenthal put forward 3 ideas related to realistic mathematics education, namely: (1) Guided Reinvention, (2) Level in the Learning, and (3) Didactical Phenomenology (Gravemeijer & Terwel, 2000). Furthermore, Treffers (Marja Van den Heuvel-Panhuizen & Paul Drijvers, 2014) put forward six operational principles of RME including:

a. Activity principle
b. Reality principle
c. Level principle
d. Interwinement principle
e. Interactivity principle
f. Guidance principle

The circle is one of the materials in the geometry domain taught to grade XI students in Indonesia. Based on the Decree of the Education Standards, Curriculum, and Assessment Agency number 8 of 2022 (Decree of BSKAP No.8, 2022) the learning outcomes in the material on circles for grade XI are that students can apply theorems about circles, and determine the length of the arc and the area of the sector of a circle to solve problems (including determining the location of the position on the Earth's surface and the distance between two places on Earth). Then the learning of circles in the sub-material of arcs and tangents to circles will be made into an iceberg learning flow. As shown in Figure 1 below.

Figure 1. Description of the Iceberg Learning Flow



According to the principle of reality in RME, learning must begin with contextual problems. So the researcher used identical rubber pipe ties to convey an understanding of the arc of a circle and the tangent line of a circle. The relationship between the learning flow and the RME principle can be seen in Table 1.

Table 1. Iceberg learning flow description

Prinsip PMRI	Konten
Activity principle	Students solve the problems given
Reality principle	Length of rubber after inserting pipe
Level principle	In accordance with class XI material
Interwinement principle	Integrated with general knowledge
Interactivity principle	Cooperation between group members, class discussions
Guidance principle	Teachers accompany students' activities

The implementation of this design research aims to form a learning trajectory in studying circles so that in studying circles it is not just a matter of memorizing formulas but using the learning trajectory that is formed to increase a deeper understanding of the material on circles that is linked to real-world situations.

Methods

In finding the learning trajectory in learning arcs and tangents to circles in the context of the length of rubber bands that bind several pipes, a design research approach will be used. Learning is carried out based on Indonesian Realistic Mathematics Education (RME) through three phases, namely: (1) preparation of experiments, (2) experiments in class, and (3) carrying out retrospective analysis (Gravemeijer & Cobb, 2006). These three phases form a cycle. In this study, two cycles will be conducted with different classes. The first cycle of HLT was tested in a small group consisting of six grade XI students at a high school in Batang district, Central Java. The main trial was conducted in a different class with a larger number of students, namely 31 students.

The following is a plan of activities carried out in this research.

Table 2. Summary of research activities

No	Fase	Kegiatan Penelitian	Pengumpuan Data
1	Preparation	Needs analysis, curriculum analysis, student character analysis, literature analysis, designing HLT, validating HLT to seven mathematics educators, revising HLT	Checklist,Document Analysis, Observation, Interview, Expert Validation
2	Experiment Implementation	Small group trial: investigating the practicality of HLT and Main group trial: investigating the practicality and effectiveness of HLT on students' mathematical literacy abilities.	Observation,Analyzing student work, Video recording, Test
3	Retrospective Analysis	Discussing and reflecting on the results of the implementation of the experimental stage to revise and improve the HLT to obtain a valid, practical, and effective learning trajectory for teaching tangents and circular arcs to improve students' mathematical literacy skills.	Document analysis

Results

The results show a picture of the learning trajectory in learning arcs and tangents to circles in the context of rubber binding several pipes. This study began with compiling a needs analysis, analyzing the curriculum, essential concepts, and student characteristics, reviewing the literature on number pattern teaching methods, and designing HLT for teaching arcs and tangents to circles with the RME approach. There are five activities involved in this HLT, where each activity involves students in solving one contextual problem. The objectives of this activity include:

1. calculate the length of the rubber binding 2 identical pipes.
2. calculate the length of the rubber binding 3 identical pipes.
3. calculate the length of the rubber binding 4 identical pipes.
4. Find a pattern for calculating the length of rubber binding for several identical pipes based on the results of activities 1, 2, and 3.
5. solve problems that are identical to activities 1, 2, and 3 by applying conclusions about calculation patterns in activity 4.

This HLT was validated by 6 mathematics teachers and 1 mathematics education lecturer who studied RME. After going through discussions with experts, HLT achieved valid criteria for both content validity and construct validity. The results of the validity stated: (1) the contextual problems presented have the potential to help students find patterns in calculating the length of rubber bands for several identical pipes; (2) the activities given to students can guide students to experience vertical and horizontal mathematization processes; (3) HLT reflects the characteristics of learning design: (4) HLT has the potential to improve students' mathematical literacy skills in learning arcs and tangents to circles.

The results of the small group trial showed that the HLT teaching arcs and tangents to circles in calculating the length of rubber ties for several identical pipes ran as expected. Students were able to solve the problems presented in the form of calculating the length of rubber ties for 2 pipes, 3 pipes, and 4 pipes correctly according to their ideas for solving them. Predicting answers prepared in advance can help students achieve learning objectives. Almost the same situation also occurred in the classroom experiment. In the classroom experiment activities, the researcher developed the material by adding activities to solve the length of rubber ties for pipes with different diameters or not identical. The next modification was that the completion of the student activity sheet was carried out in groups with a maximum of 4 students.

At the beginning of the activity, the teacher provides a situational description of the context that will be faced. The situational context is as follows.

Figure 2. Situational Context

A pipe company will send pipes in a way that is packed for several pipes into 1 bundle. According to consumer demand, the pipes will be packed with three types of pipe packaging, namely:

- 2 pipe packaging
- 3 pipe packaging
- 4 pipe packaging

The pipe is tied using a special rubber designed with a certain elasticity that is maximally filled according to the type of packaging. If the pipes that are packed are identical with a radius of 14 cm, then we will determine the maximum length of the rubber after the pipe is inserted.

Below we will calculate the maximum length of rubber after the pipe is inserted through the activities in math assignment 1, math assignment 2, and math assignment 3.

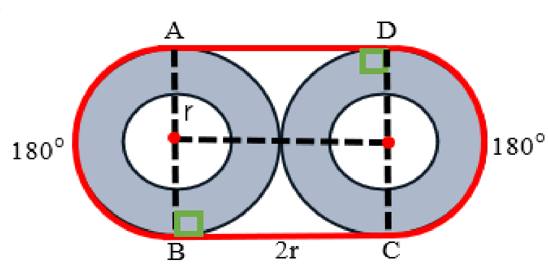
After the situational context is delivered, the designed activities are implemented. The implementation of activities in cycle 1 is described as follows.

Math Task 1: Calculating the Length of the Rubber Band Binding 2 Identical Pipes

In this first activity, a problem is presented to calculate the length of the rubber binding two pipes. The problem of activity 1 is as follows.

Figure 3. Mathematics task in activity 1

Determine the maximum length of the rubber after inserting 2 pipes. The illustration is as follows



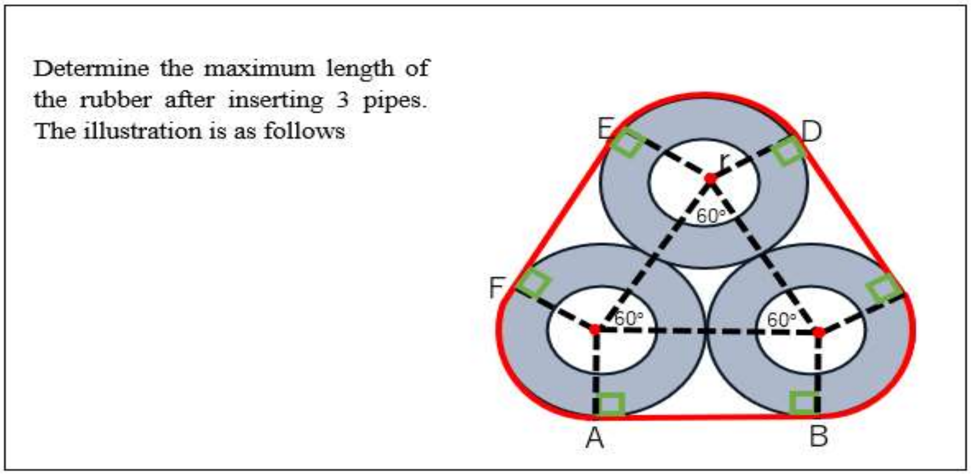
The activity begins by giving students time to complete Activity 1 in various ways, both formal and informal, according to their knowledge. Most students start by describing the picture, giving symbols to the points formed, and writing the rubber part with the symbol r, with r being the radius of the circle. Then on the rubber part that is in the form of a 180 ° arc, students symbolize it as a semicircle. Thus, 2 rubber parts are formed in the form of a semicircular arc. Then the last step is for students to add up the length of all the rubber.

The steps taken by students are in accordance with one of the answers made by the researcher in HLT. At the discussion stage, the teacher asks students to simplify the algebraic form of the calculation.

Math Task 2: Calculating the Length of the Rubber Band Binding 3 Identical Pipes

In this second activity, a problem is presented that is almost the same as activity 1 to calculate the length of the rubber binding three identical pipes. The problem of activity 2 is as follows.

Figure 4. Mathematics Task in Activity 2

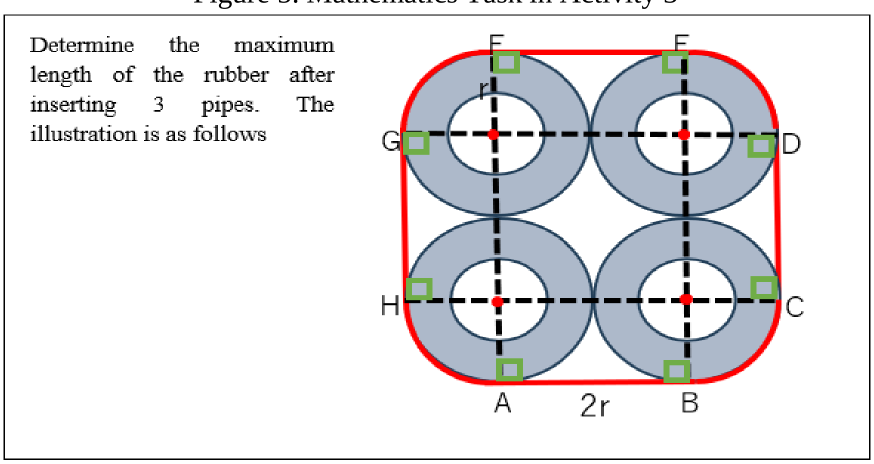


In activity 2, students work on mathematics assignment 2 in the same way as in activity 1. In the discussion stage, students are asked to simplify the algebraic form that is formed.

Math Activity 3: Calculating the Length of the Rubber Band Binding 4 Identical Pipes

In this third activity, a problem is presented that is almost the same as activity 1 to calculate the length of the rubber binding four identical pipes. The problem of activity 1 is as follows.

Figure 5. Mathematics Task in Activity 3



In activity 3, students work on mathematics assignment 3 in the same way as in activities 1 and 2. In the discussion stage, students are asked to simplify the algebraic forms that are formed.

Activity 4: Finding a Pattern for Calculating the Length of the Rubber Band Based on the Results of the Previous 3 Activities

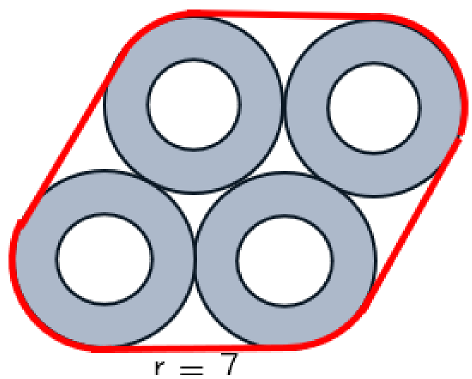
At the stage of drawing conclusions, students are asked to observe the answers in activity 1, activity 2, and activity 3. From the three answers given, students are asked to draw conclusions. After all students have concluded, the teacher holds a discussion by asking students to convey their respective conclusions. Then the teacher provides reinforcement to the conclusions given by each student.

Activity 5 Solve Problems That Are Identical to the Previous Activity

In the fifth activity, students are given practice questions that are identical to the problems in activities 1, 2, and 3. The practice questions are as follows.

Figure 6. Practice questions in activity 5

A pipe company will send pipes by packing 4 pipes into 1 package tied with rubber. The company has designed rubber with a certain elasticity that can be filled by a maximum of 4 pipes. If the four pipes are identical with a radius of 7 cm, then determine the maximum length of the rubber after five pipes are inserted.



In this activity, students are asked to work on the questions by utilizing the knowledge obtained previously, namely in activities 1, 2, 3, and 4. The results of working on the activity sheets in cycle 1 can be summarized in the following table.

Table 3. Summary of student answers on the student activity sheet

Student Name	TM1	TM2	TM3	Conclusion	Exercises
Subject 1	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	Any number of curves totals 1 circumference of the circle	MP,PB,MJ
Subject 2	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The arc forms 1 circumference of the circle	MP,PB,MJ
Subject 3	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	Even though the ropes tied are different, it will still produce 360° or 1 circle.	MP,PB,MJ
Subject 4	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	From the previous activity, I can conclude that the angle formed is 360°	MP,PB,MJT
Subject 5	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	Even though the circles are different, the k remains the same / result is 1	MP,TJ
Subject 6	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	So, all the angles of the arc/curved line, no matter how many circles there are, still add up to 1 k (1 circumference of the circle).	MP,PB,MJ

Table 4. Description of symbols in table 3

Code	Description
MP	Breaking down the length of the rubber into parts consisting of external common tangents (circles) and arc parts.
PB	Determine the size of the arc by comparing it to the circumference of the circle.
MJT	Calculate the total amount (with formula) correctly
MJ	Calculating the total amount, but there was an error in the calculation
TJ	Doesn't give a clear answer

The results show that all students completed activities 1, 2 and 3 by breaking down the length of the rubber into parts in the form of external common tangents (circles) and arc parts, then determining the size of the arc by comparing it with the circumference of the circle, and all students succeeded in calculating the length of all the rubber ties correctly.

In addition, most students concluded that the total arc size formed was 360° or formed one circumference of a circle. In the results of the discussion, the teacher and students concluded that the length of the rubber band binding several identical pipes can be formulated as.

Length of rubber = $2nr + 2\pi r$

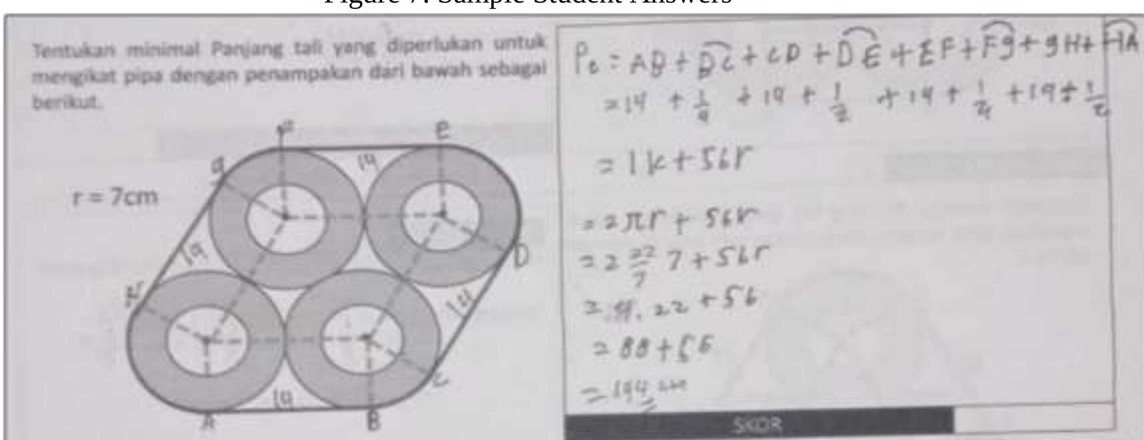
Or

Length of rubber = $2r(n + \pi)$

With *r* being the radius of identical pipes and *n* is the number of pipes.

In the results of the practice questions, it can be seen that students completed the questions with the stages carried out in activities 1, 2, and 3. There was 1 student who managed to complete the calculation correctly. The following is the subject's answer.

Figure 7. Sample Student Answers



Based on the answer, the subject has completed the practice questions using the conclusions obtained in activity 4. The student began the answer by partitioning the rubber part into a straight part and an arc part. Then continued by adding the two parts. From the answer, it can also be seen that the student succeeded in simplifying the number of arcs into the circumference of a circle. The error occurred in the description of the arc where the subject said that the size of each arc was a quarter of a circle. However, the results obtained were correct because the subject used the formula obtained in the discussion activity.

After cycle I was implemented, the researcher conducted a retrospective analysis. Based on the results of the retrospective analysis, the researcher concluded several things related to learning in cycle I. These conclusions include: (1) Cycle I learning activities are in accordance with the HLT that has been prepared previously, (2) subjects successfully carried out learning activities well, (3) subjects and teachers successfully found a formula to calculate the length of rubber ties for several pipes. From the results of the retrospective analysis, the researcher made modifications to the methods and tools that would be used in cycle II. In cycle II, the activity sheets were worked on in groups with a maximum of 4 students. Considering the greater number of personnel and longer learning time, the researcher added 1 learning activity. Thus, there will be two learning activities in cycle II. Activity 1 is intended to calculate the length of rubber ties for several identical pipes and learning activity 2 is intended to determine the length of rubber ties for several pipes of different sizes (not identical).

In learning activity 2, subjects completed mathematics tasks 1, 2, and 3, drew conclusions, and practiced questions with the same instrument as in cycle 1. The results of completing the student activity sheet in Learning 1 show that the learning activities are by HLT. The completion stages in activities 1, 2, and 3 are in accordance with HLT, students provide conclusions in accordance with HLT, and in the results of the discussion, students are successfully directed by the teacher to find the formula for the length of the connecting train for several pipes, namely.

Length of rubber = $2nr + 2\pi r$

The results of the practice questions showed that 62.5% of the groups had successfully completed the questions correctly and 37.5% had completed them in the correct manner but there were errors in the calculation process.

Table 5. Summary of student answers in Studen Activity Sheet Activity 1

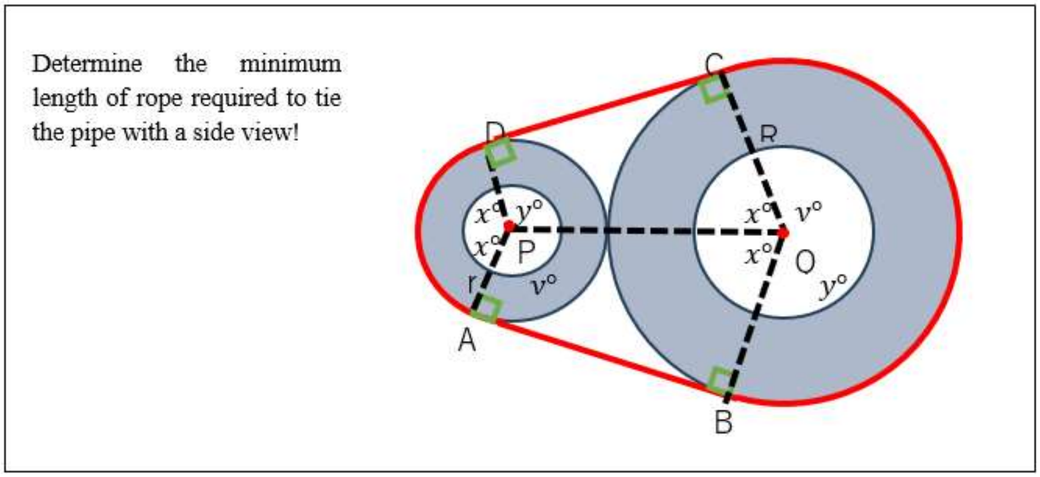
Group	TM1	TM2	TM3	Conclusion	Excercise
Group 1	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The length of all curved lines is equal to 1 circumference of the circle	MP,PB,MJ
Group 2	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The entire arc is 360°	MP,PB,MJT
Group 3	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	Length of rubber 2nr plus the length of the entire bow	MP,PB,MJ
Group 4	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The length of the arc always forms a full circle, namely 360° and the length of the straight side can be calculated by 2 times the number of pipes times r.	MP,PB,MJT
Group 5	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The length of a curved line is equal to one circumference of a circle, which is equal to $2\pi r$	MP,PB,MJT
Group 6	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The length of the rubber is obtained by adding the length of all the straight sides and all the curved sides.	MP,PB,MJ
Group 7	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The length of the arc is equal to one circumference of the circle	MP,PB,MJT
Group 8	MP,PB, MJT	MP,PB, MJT	MP,PB, MJT	The length of an arc is equal to one circumference of a circle with an arc size of 360°	MP,PB,MJT

From learning activity 1, the activity is then continued in learning activity 2. Learning activity 2 can be described as follows.

Math Assignment 1

In math assignment 1, a problem is presented in the form of two pipes with different radius sizes tied with rubber. Then students are asked to determine the length of the rubber after both pipes are inserted into it. The problem in math assignment 1 is as follows.

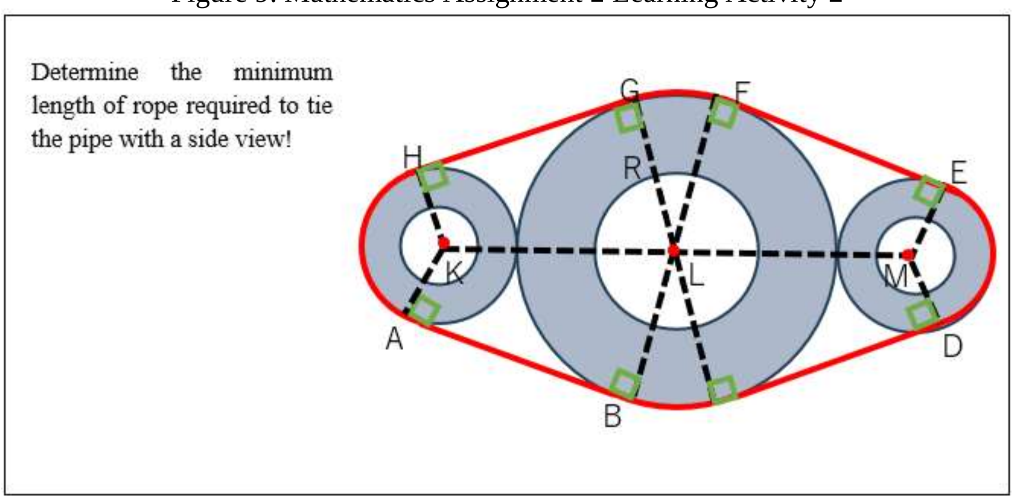
Figure 8. Mathematics Task 1 Learning Activity 2



Math Assignment 2

In math assignment 2, a problem is presented in the form of three pipes with 2 pipes of the same size and one pipe of different radius sizes tied with rubber. Then students are asked to determine the length of the rubber after the three pipes are inserted into it. The problem of math assignment 2 is as follows.

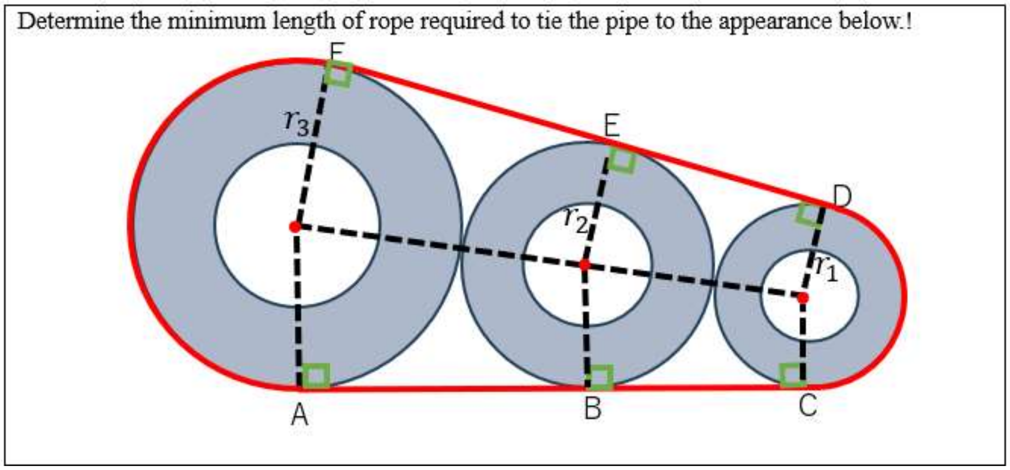
Figure 9. Mathematics Assignment 2 Learning Activity 2



Math Assignment 3

In math assignment 2, a problem is presented in the form of three pipes with 3 different-sized pipes tied with rubber. Then students are asked to determine the length of the rubber after the three pipes are inserted into it. The problem of math assignment 3 is as follows.

Figure 9. Mathematics Task 3 Learning Activity 2



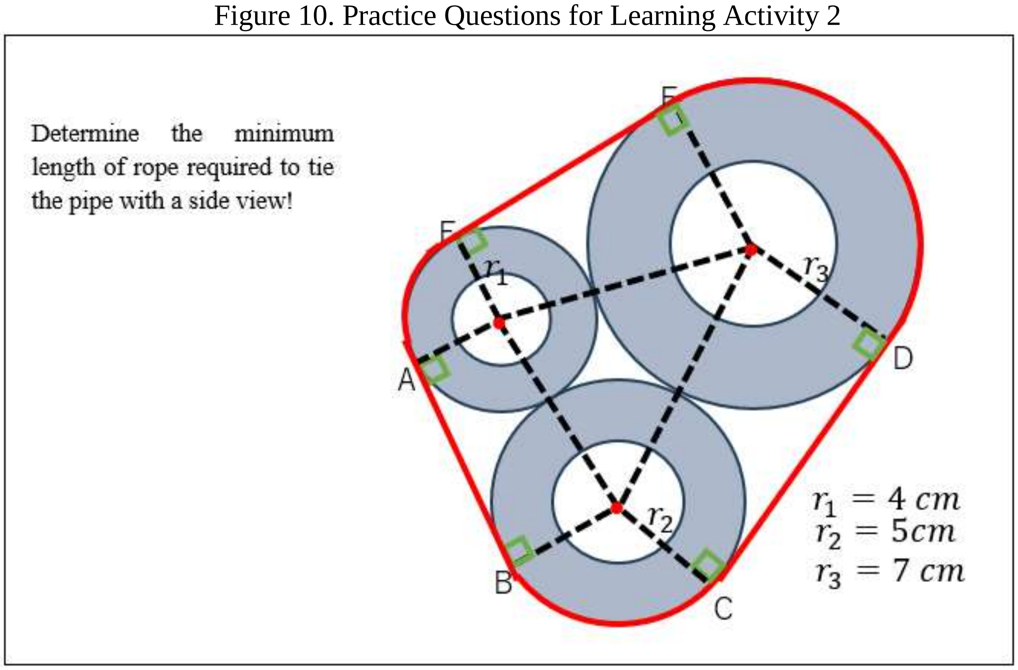
Draw a Conclusion

At the conclusion stage, students are asked to observe the answers to the three activities carried out, namely activity 1, activity 2, and activity 3. From the answers given by the students in the three activities, they were asked to make conclusions. After all students have created a conclusion, the teacher will hold a discussion where students are asked to convey their respective conclusions. Furthermore, the teacher will provide reinforcement to the conclusions that have been submitted by each student.

Exercises

In the fifth activity, students are given practice questions that are identical to the problems in activities 1, 2, and 3. The practice questions are as follows.

Figure 10. Practice Questions for Learning Activity 2



From the five activities that have been carried out in Learning 2, the results obtained for completing the student activity sheets in Learning 2 are as follows.

Table 6. Summary of student answers on Student Activity Sheet Activity 2

Group Name	TM1	TM2	TM3	Conclusion	Exercises
Group 1	MP	MP	MP	The entire arc is 360°	MP,MJ
Group 2	MP	MP	MP	Large rubber that curves 360°	MP,MJ
Group 3	MP	MP	MP	The curved rubber part is 360° in size	MP,MJ
Group 4	MP	MP	MP	The central angle of the entire arc is 360°	MP,MJ
Group 5	MP	MP	MP	The length of the arc is calculated according to the size of the angle and the radius of each circle.	MP,MJ
Group 6	MP	MP	MP	The length of the rubber cannot be formulated specifically	MP,MJ
Group 7	MP	MP	MP	There is no discernible pattern	MP,MJ
Group 8	MP	MP	MP	The length of the rubber binding is calculated manually one by one.	MP,MJ

The results of completing the student activity sheet in Learning 2 show that students complete math tasks 1, 2, and 3 by breaking down the length of the rubber into parts in the form of external common tangents (circles) and arc parts. The results show that students have difficulty in calculating the length of the rubber. The difficulty is because the different pipe sizes cause students to have to calculate one by one the length of the Outer alliance tangent and the arcs formed. Until the time the work is finished, students can only write down the formula for calculating.

The results of the conclusion drawing activity can be concluded that the size of the central angle for all the arcs formed is 360° and the length of the rubber pipe ties that are not identical cannot be formulated specifically.

The results of the practice questions show that there are difficulties in calculating the length of the rubber binding several pipes that are not identical. This can be seen in the calculation process that has not been completed and no one has answered correctly, but all groups have implemented the solution to the problem by calculating the length of the rubber in the form of outer alliance tangent and the one in the form of an arc.

From all the activities that have been carried out, it shows that mathematics can be used to solve problems in everyday life. This is in accordance with Hans Freudenthal's view, namely "Mathematics Should be Connected to Reality"(Zulkardi, 2002). The context of tying several pipes as previously stated is a problem that is often encountered in everyday life. This context is not only limited to pipe packaging, but can be used for similar contexts, for example tying identical logs, tying broomsticks, tying culverts or electricity poles into transport trucks, and many others.

The results of the study show that the length of the identical pipe-binding rubber can be formulated into a simple formula, namely $2nr + 2\pi r$. While non-identical pipes, cannot be formulated specifically, however, both the length of the identical and non-identical pipe binding rubber in the arc section form a central angle whose total central angle is 360°.

The formula for the length of rubber binding identical pipes is obtained by the following learning path: (1) calculating the length of rubber binding two pipes; (2) calculating the length of rubber binding three pipes; (3) calculating the length of rubber binding four pipes; (4) drawing conclusions; and (5) practicing questions. This formula can be applied to calculate similar cases, for example in calculating the length of rope binding identically sized wood, culverts, electric poles, and many others.

Discussion

From all the activities that have been carried out, it shows that mathematics can be used to solve problems in everyday life. This is in accordance with Hans Freudenthal's view, namely "Mathematics Should be Connected to Reality"(Zulkardi, 2002). Learning with the RME approach can help teachers to simplify and realize mathematical concepts (Laurens et al., 2018). The context of tying several pipes as previously stated is a problem that is often encountered in everyday life. We can find contexts similar to the context of rubber lengths to tie pipes in our lives such as rope lengths to tie logs, tie culverts, or tie electric poles to the side of the road on transport trucks. These events are events that we often encounter in our daily lives. Learning with the RME approach can increase the emergence of effective mathematical practice and develop conceptual understanding (Gökkurt Özdemir, 2017).

In building an understanding of the concept, it begins with an understanding of the process of calculating the length of rubber used in the packaging of 2 pipes, 3 pipes, and 4 pipes. From the results of the calculation, observations are made of the calculation process, symbols for the variables used, and simplification of the algebraic forms obtained so that simple formulas are formed. In the last stage, similar or modified questions were practiced from the questions in activities 1, 2, and 3. The practice questions are completed using the formula or conclusion obtained in the previous stage. By solving using the formula obtained, it will provide an overview of how to solve with mathematical calculation methods and solve using formulas.

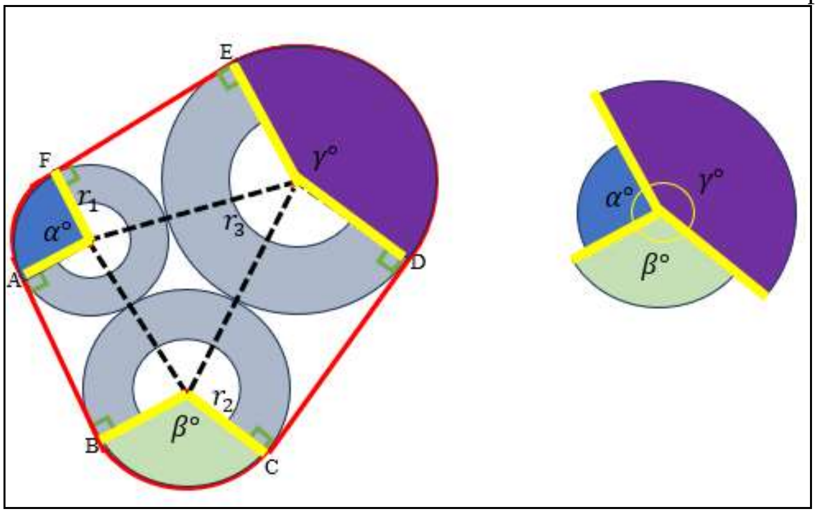
Activities 1,2, 3, and 4 on the learning trajectory provide basic knowledge of how a formula is formed. This basic knowledge is an important concept as it is the basis for calculating the length of the rubber binder for various pipe packaging cases. At this stage, students can gain knowledge of formula formation with their own knowledge through the activities carried out.

Meanwhile, the 5th activity provides an overview of how to use a simple formula that can make the work easier in calculating the length of rubber used in tying identical pipes. This stage can be a tester for the use of the formula formed. Is the formula applicable to calculate the length of the rubber binding for various variations in pipe arrangement and variations in identical pipe sizes.

From a series of activities in this learning track, conceptual knowledge can be fostered towards the formation of a formula. Not only getting formulas to memorize but knowledge of the formation of formulas has also been obtained. Thus, students are not dependent on formulas, they can solve problems related to the length of the binding rubber in pipe packaging with various packaging variations and whatever the number of pipes. They will not worry if they forget the formula, they can even form the formula themselves.

The results of the study show that the length of the identical pipe-binding rubber can be formulated into a simple formula, namely $2nr + 2\pi r$. While non-identical pipes, cannot be formulated specifically, however, both the length of the identical and non-identical pipe binding rubber in the arc section form a central angle whose total central angle is 360°.

Figure 11. The size of the arc section on the rubber fastener of the non-identical pipe is 360°



The formula for the length of rubber binding identical pipes is obtained by the following learning path: (1) calculating the length of rubber binding two pipes; (2) calculating the length of rubber binding three pipes; (3) calculating the length of rubber binding four pipes; (4) drawing conclusions; and (5) practicing questions. This learning track can be an alternative way for teachers to design a series of learning activities in circle learning, especially on arc meters and circle tangent lines.

Conclusion

The learning trajectory that is formed can be practiced for similar contexts that exist in everyday life, namely to calculate the length of rubber or rope binding cylindrical objects with identical circular surfaces. Students can more easily calculate the length of rubber binding identical pipes by using the formula found, namely $2nr + 2\pi r$, where n is the number of pipes and r is the radius of the identical pipe

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